MAT 102: Ordinary Differential Equations

Topic 3: Second Order Linear Ordinary Differential Equations

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1 Tutorial Questions — Topic 3

1.1 Section A: Linear Independence and the Wronskian

Q1. Compute the Wronskian of each pair of functions and determine whether they are linearly independent:

(a) $y_1 = e^{3x}$, $y_2 = e^{-x}$

(b) $y_1 = \cos 3x$, $y_2 = \sin 3x$

(c) $y_1 = x^2$, $y_2 = x^3$

(d) $y_1 = e^x$, $y_2 = e^{x+2}$

(e) $y_1 = e^{2x}\cos x$, $y_2 = e^{2x}\sin x$

(f) $y_1 = 1 + x$, $y_2 = 1 - x$

Q2. Verify that y_1 and y_2 are solutions of the given ODE, compute their Wronskian, and write the general solution.

(a) $y_1 = e^{-2x}$, $y_2 = e^{3x}$; ODE: $y'' - y' - 6y = 0$

(b) $y_1 = x$, $y_2 = xe^x$; ODE: $x^2y'' - (x^2 + 2x)y' + (x + 2)y = 0$, $x > 0$

(c) $y_1 = e^x \cos 2x$, $y_2 = e^x \sin 2x$; ODE: $y'' - 2y' + 5y = 0$

1.2 Section B: Homogeneous Equations — Constant Coefficients

Q3. Solve the following homogeneous ODEs:

(a) $y'' - 7y' + 12y = 0$

(b) $y'' + 6y' + 9y = 0$

(c) $y'' - 4y' + 13y = 0$

(d) $y'' - 9y = 0$

(e) $y'' + 2y' + y = 0$

(f) $4y'' - 4y' + y = 0$

(g) $y'' + \omega^2 y = 0$ (simple harmonic oscillator, $\omega > 0$)

(h) $y'' + 2y' - 8y = 0$

Q4. Solve the IVPs:

(a) $y'' + 4y = 0, \quad y(0) = 3,\ y'(0) = -2$

(b) $y'' - 2y' - 3y = 0, \quad y(0) = 2,\ y'(0) = -2$

(c) $y'' + 6y' + 9y = 0, \quad y(0) = 2,\ y'(0) = 1$

(d) $y'' - 4y' + 5y = 0, \quad y(0) = 1,\ y'(0) = 0$

(e) $y'' - 6y' + 9y = 0, \quad y(0) = 0,\ y'(0) = 2$

1.3 Section C: Method of Undetermined Coefficients

Q5. Find a particular solution y_p using undetermined coefficients:

(a) $y'' - y' - 2y = 3e^{2x}$

(b) $y'' + 4y = 2\cos x$

(c) $y'' + 2y' - 3y = x^2$

(d) $y'' + y' = 2x^2 - 1$

(e) $y'' - y = e^x \sin x$

(f) $y'' + y = \sin x$ *(watch for modification rule)*

(g) $y'' - 2y' = 4x + 2$

Q6. Solve the following non-homogeneous ODEs completely (find $y_h + y_p$):

(a) $y'' - 5y' + 6y = 2e^x$

(b) $y'' + 4y' + 4y = 3xe^{-2x}$ *(modification rule applies!)*

(c) $y'' - 4y = 4x^2 - 2$

(d) $y'' + 3y' + 2y = \cos x + \sin x$

(e) $y'' - y = e^x + e^{-x}$ *(modification rule applies twice!)*

1.4 Section D: Variation of Parameters

Q7. Use variation of parameters to find the general solution:

(a) $y'' + y = \tan x$

(b) $y'' + 4y = 4\csc 2x$

(c) $y'' - 2y' + y = \dfrac{e^x}{x^2}, \quad x > 0$

(d) $y'' - y = \dfrac{2}{1 + e^x}$

(e) $y'' + 3y' + 2y = \dfrac{1}{1 + e^x}$

Q8. Solve the following IVPs using variation of parameters:

(a) $y'' - y = 2e^x, \quad y(0) = 0,\ y'(0) = 1$

(b) $y'' + 4y = \sec 2x, \quad y(0) = 0,\ y'(0) = 0$

1.5 Section E: Initial and Boundary Value Problems

Q9. Solve the IVPs completely:

(a) $y'' - 3y' + 2y = 4e^x, \quad y(0) = 1,\ y'(0) = 1$

(b) $y'' + 4y' - 5y = 3e^{2x}, \quad y(0) = 2,\ y'(0) = 1$

(c) $y'' + y = x\cos x, \quad y(0) = 0,\ y'(0) = 1$ *(Hint: use undetermined coefficients — modification needed)*

Q10. For each BVP, determine whether there is no solution, a unique solution, or infinitely many solutions. Find the solution if it exists.

(a) $y'' + y = 0, \quad y(0) = 0,\ y(\pi) = 0$

(b) $y'' + y = 0, \quad y(0) = 1,\ y'(\pi) = 0$

(c) $y'' + 4y = 0, \quad y(0) = 0,\ y(\pi) = 2$

(d) $y'' + 4y = 0, \quad y(0) = 0,\ y(\pi/2) = 0$

(e) $y'' + 9y = 0, \quad y(0) = 0,\ y(\pi/3) = 1$