Topic 4 — Laplace Transforms and Applications to Differential Equations

Tutorial Questions

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Instructions

Show **all** working at every step.

1 Section A: Improper Integrals (Revision)

Q1. Evaluate the following improper integrals. Show all limit working.

(a) $\int_0^\infty e^{-4t}\,dt$

(b) $\int_0^\infty te^{-st}\,dt$ (assume $s > 0$) — this is the definition calculation for $\mathcal{L}\{t\}$

(c) $\int_0^\infty e^{-3t}\cos 2t\,dt$ *(Hint: use* $\int e^{at}\cos bt\,dt = \frac{e^{at}(a\cos bt + b\sin bt)}{a^2+b^2}$)

(d) Does $\int_0^\infty e^{2t}\,dt$ converge or diverge? Explain.

2 Section B: Laplace Transforms from Definition and Table

Q2. Use the definition to prove $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$ for $s > a$.

Q3. Use the table and linearity. State which row at each step.

(a) $f(t) = 6t^2 - 3t + 5$

(b) $f(t) = 4e^{-2t} - 3\sin 4t + 2\cos t$

(c) $f(t) = (t+3)^2$ *(expand first)*

(d) $f(t) = e^{5t}\cos 3t$ *(First Shifting Theorem)*

(e) $f(t) = t^3 e^{-2t}$ *(First Shifting Theorem)*

(f) $f(t) = 2e^{-t}\sin 5t$

(g) $f(t) = 5\cosh 2t - 3\sinh 2t$ *(Rows 7 and 8)*

(h) $f(t) = (2e^t - 3)^2$ *(expand first)*

3 Section C: Inverse Laplace Transforms

Q4. Find $\mathcal{L}^{-1}\{F(s)\}$ directly from the table (no partial fractions needed):

(a) $\dfrac{6}{s^4}$

(b) $\dfrac{4}{s+7}$

(c) $\dfrac{3}{s^2+16}$

(d) $\dfrac{s}{s^2+9}$

(e) $\dfrac{2}{(s-3)^2+4}$

(f) $\dfrac{s+1}{(s+1)^2+9}$

Q5. Partial fractions — distinct linear factors:

(a) $\mathcal{L}^{-1}\left\{\frac{5}{(s+1)(s-2)}\right\}$

(b) $\mathcal{L}^{-1}\left\{\frac{2s+3}{(s+2)(s-1)}\right\}$

(c) $\mathcal{L}^{-1}\left\{\frac{s}{(s-1)(s+2)(s+3)}\right\}$

Q6. Partial fractions — repeated factors:

(a) $\mathcal{L}^{-1}\left\{\frac{3}{s^2(s+3)}\right\}$

(b) $\mathcal{L}^{-1}\left\{\frac{s+2}{(s+1)^2(s-1)}\right\}$

Q7. Partial fractions and completing the square:

(a) $\mathcal{L}^{-1}\left\{\frac{5}{s^2+6s+10}\right\}$

(b) $\mathcal{L}^{-1}\left\{\frac{3s+1}{s^2+4s+5}\right\}$

(c) $\mathcal{L}^{-1}\left\{\frac{2s}{(s+1)(s^2+2s+5)}\right\}$

4 Section D: Transform of Derivatives — Practice

Q8. Write $\mathcal{L}\{y''\}$ and $\mathcal{L}\{y'\}$ in terms of $Y(s)$:

(a) $y(0) = 0,\ y'(0) = 0$

(b) $y(0) = 4,\ y'(0) = -3$

(c) $y(0) = -1,\ y'(0) = 2$

5 Section E: Solving First-Order IVPs

Q9. Solve using Laplace transforms. Show all four steps.

(a) $y' - 2y = 0,\ y(0) = 5$

(b) $y' + 3y = 6,\ y(0) = 0$

(c) $y' + y = e^{2t},\ y(0) = 2$

(d) $y' + 4y = 8t,\ y(0) = 1$

(e) $y' - y = \sin t,\ y(0) = 0$

6 Section F: Solving Second-Order IVPs

Q10. Solve using Laplace transforms.

(a) $y'' - y' - 6y = 0,\ y(0) = 1,\ y'(0) = 1$

(b) $y'' + 4y = 0,\ y(0) = 0,\ y'(0) = 3$

(c) $y'' + 6y' + 9y = 0,\ y(0) = 1,\ y'(0) = 0$

(d) $y'' + 4y' + 4y = e^{-2t},\ y(0) = 0,\ y'(0) = 0$

(e) $y'' - 4y = 8t,\ y(0) = 0,\ y'(0) = 0$

7 Section G: Simultaneous Differential Equations

Q11. Solve each system using Laplace transforms.

(a)

$$x' = 4x - y, \quad y' = 2x + y, \quad x(0) = 3,\ y(0) = 0$$

(b)

$$x' - y = 0, \quad y' + x = 1, \quad x(0) = 0,\ y(0) = 0$$

(c)

$$x' + x + y = 0, \quad y' + x + y = e^t, \quad x(0) = 1,\ y(0) = 0$$