

# QMS 101 Introductory Statistics

## Topic V: Permutations and Combinations

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# Learning Outcomes

By the end of this topic, students will be able to:

- ▶ Explain why counting methods are needed in statistics
- ▶ Compute factorials and apply the sum and product rules
- ▶ **Derive** the permutation formula from first principles
- ▶ **Derive** the combination formula from the permutation formula
- ▶ Distinguish between permutations and combinations and choose correctly
- ▶ Apply both formulas to business, agricultural, and social problems

# The Bridge from Topic IV to This Topic

## Why Counting Comes Before Probability

In **Topic VI** (Probability Theory), we will calculate the chance of an event occurring:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

Both the numerator and denominator require us to **count** — often for very large numbers of possibilities.

For example: *“What is the probability that a committee of 3 randomly selected from 10 farmers includes the two most experienced farmers?”*

To answer this, we must first know: - How many committees of 3 can be formed from 10? (*counting*) - How many of those include the two specific farmers? (*counting*)

**Permutations and combinations are the tools for that counting.**

# Our Running Context — Farm Management Scenarios

## The Setting

The farm manager from our earlier topics has been asked by the district agricultural office to help with several organisational and planning tasks.

Each task requires counting the number of possible arrangements or selections — and the numbers grow so large so quickly that we need systematic formulas.

We will use the **same farm management context** throughout this topic, introducing new scenarios as each concept is developed.

## 5.1 Motivation — Why Counting Matters

# The Counting Problem

## Scenario 1 — Scheduling Farm Visits

The district agricultural officer must visit **4 farms** (Farm A, B, C, D) in a single day.

In how many different orders can she schedule the visits?

She could try listing them all:

*ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC*

This is already getting long — and there are only 4 farms.

**What if there were 10 farms? Or 20?**



Listing every possibility is impractical for large problems.  
We need a **formula** that counts without listing.

# How Fast Numbers Grow

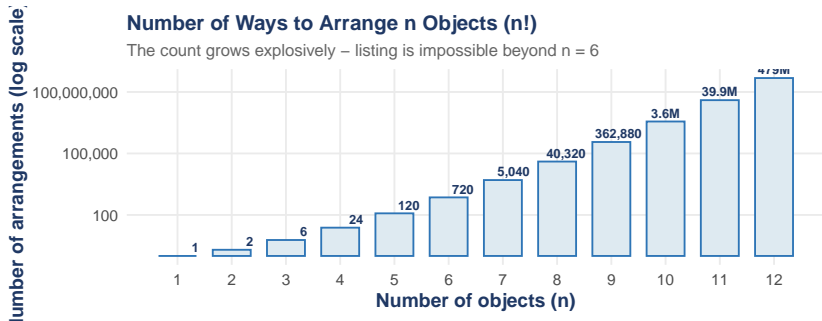


Figure 1

4 farms → **24** orderings | 10 farms → **3,628,800** | 20 farms → **2.4 quintillion**

Clearly we need a formula, not a list.

## 5.2 Factorials

## Building the Factorial Idea

How Many Ways to Seat 3 Farmers?

Three farmers — Amina (A), Baraka (B), Chakula (C) — sit in a row of 3 chairs.

**Chair 1** can be filled by any of the **3** farmers.

Once seated, **Chair 2** can be filled by any of the remaining **2**.

**Chair 3** must be filled by the **1** remaining farmer.

$$\text{Total arrangements} = 3 \times 2 \times 1 = 6$$

List to verify: ABC, ACB, BAC, BCA, CAB, CBA — exactly 6.

**Generalising:** For  $n$  distinct objects arranged in a row:

$$\underbrace{n}_{\text{1st place}} \times \underbrace{(n-1)}_{\text{2nd place}} \times \underbrace{(n-2)}_{\text{3rd place}} \times \cdots \times \underbrace{2} \times \underbrace{1}_{\text{last place}}$$

## Defining the Factorial

$$n! = n \times (n - 1) \times (n - 2) \times \dots \times 2 \times 1$$

### Definition

$n!$  (read: “ $n$  factorial”) is the product of all positive integers from  $n$  down to 1.

It counts the number of ways to arrange  $n$  distinct objects in a line.

### Key values to memorise:

$n$	$n!$	$n$	$n!$
0	<b>1</b> (special case)	5	120
1	1	6	720
2	2	7	5,040
3	6	10	3,628,800
4	24	12	479,001,600

## Simplifying Factorial Expressions

A very useful property: many factorials cancel in fractions.

$$\frac{8!}{5!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = 8 \times 7 \times 6 = 336$$

**Rule:**  $\frac{n!}{(n-r)!} = n \times (n-1) \times \dots \times (n-r+1)$  — the top  $r$  terms only.

### Farm Example

The officer visits 3 farms out of 8, in order. How many ordered schedules are possible?

$$\frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \times 7 \times 6 = \mathbf{336}$$

## Practice 5.1 — Factorials

### Your Turn

(a) Calculate without a calculator:

$$(i) 6! \quad (ii) \frac{9!}{6!} \quad (iii) \frac{7!}{4! \times 3!} \quad (iv) \frac{5!}{0! \times 5!}$$

(b) A village cooperative has **5 committee positions** to be filled in a meeting.

All 5 will be seated in a row for a photograph.

In how many different orders can the 5 members be arranged?

(c) The same cooperative adds a 6th member.

Without calculating  $6!$  directly, use the relationship  $6! = 6 \times 5!$  to find the answer.

(d) Explain in one sentence why  $0! = 1$  makes logical sense.

(Hint: In how many ways can you arrange zero objects?)

## 5.3 The Fundamental Counting Rules

## Two Rules for Counting Combined Events

Before deriving permutation and combination formulas, we need two simple but powerful counting rules.

# The Sum Rule (Addition Principle)

## The Sum Rule

If task A can be done in  $m$  ways **OR** task B can be done in  $n$  ways (and the two tasks are mutually exclusive — doing one excludes the other), then there are  $m + n$  ways to do **one of** the tasks.

## Warning

The sum rule applies only when the choices are **mutually exclusive** — you cannot combine them. If you could go road **AND** motorbike, you would need the product rule.

## Note

### Everyday Example: Buying a Drink

You want to buy **one** drink. The shop sells **4 types of soda** and **3 types of juice**. Since you are only buying **one** soda **OR one** juice, you add the options:

Total Choices = 4 sodas + 3 juices = 7 ways to choose

## Farm Example

The farm manager needs to travel from the district office to the farm.

She can go **by road** (3 possible routes) **OR by motorbike path** (2 possible paths).

The two modes are separate — she takes exactly one.

$$\text{Total travel options} = 3 + 2 = 5$$

# The Product Rule (Multiplication Principle)

## The Product Rule

If task A can be done in  $m$  ways **AND** task B can then be done in  $n$  ways (independently), then both tasks together can be done in  $m \times n$  ways.

## Farm Example

A seed supplier offers **2 varieties of maize** and **3 varieties of beans**.

A farmer chooses one maize variety **AND** one bean variety to plant this season.

$$\text{Total planting combinations} = 2 \times 3 = 6$$

# Visualising the Product Rule — A Tree Diagram

## Tree Diagram: 2 Maize × 3 Bean Varieties = 6 Combinations

Product Rule: each branch multiplies the choices

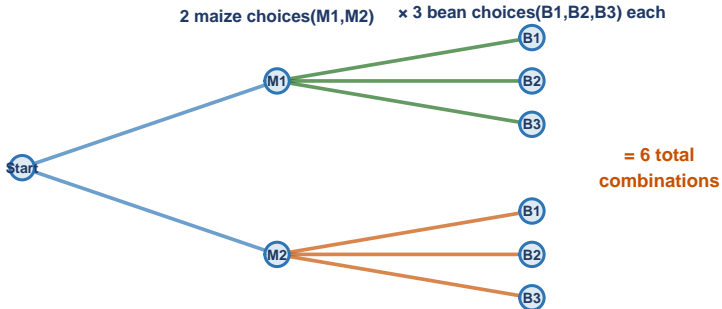


Figure 2

## Sum Rule vs Product Rule — The Key Question

### Use the **Sum Rule** when:

The tasks are alternatives — you do **one OR the other**.

*Key word: OR*

$$m + n$$

### Use the **Product Rule** when:

The tasks happen together — you do **one AND the other**.

*Key word: AND*

$$m \times n$$

### Farm Example Combining Both Rules

A farmer makes a crop choice: maize OR beans. Then, independently, chooses to irrigate OR not irrigate.

Number of distinct crop–irrigation decisions:

**Step 1** (crop choice — OR):  $4 + 3 = 7$  crop options

**Step 2** (irrigation — AND): each crop option  $\times$  2 irrigation choices

$$\text{Total} = 7 \times 2 = 14$$

## Practice 5.2 — Counting Rules

### Your Turn

**(a)** A cooperative store sells **3 types of fertiliser** and **5 types of pesticide**.

A farmer buys one fertiliser AND one pesticide. How many different purchases are possible?

**(b)** The same store also sells **4 types of seeds**.

A second farmer needs either a fertiliser OR seeds (not both). How many product choices does this farmer have?

**(c)** A farm record form requires: - A field code: letters A, B, C (3 options) - A season code: 1, 2 (2 options) - A crop code: W (wheat), M (maize), B (beans) (3 options)

How many distinct farm records can be created?

**(d)** A village of 120 farmers is divided into 4 groups.

Each group selects 1 representative independently.

How many different sets of 4 representatives are possible?

*(Use the product rule. Assume groups have 30 farmers each.)*

## Quick Guide

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	<b>Sum Rule</b>	<b>Product Rule</b>
<b>When</b>	Choices are <b>alternatives</b>	Choices are <b>sequential/combined</b>
<b>Key word</b>	<b>OR</b>	<b>AND</b>
<b>Question pattern</b>	“Choose <i>one</i> from group A <i>or</i> group B”	“Choose from A <i>and then</i> from B”
<b>Logic</b>	You do <b>one task only</b>	You do <b>multiple tasks together</b>
<b>Formula</b>	$m + n$	$m \times n$
<b>Example</b>	Buy a soda OR a juice	Buy a meal AND a drink

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## 5.4 Permutations

## Deriving the Permutation Formula

### The Problem

There are **8 farmers** at a cooperative meeting. **3 chairs** are labelled: Chairperson, Secretary, Treasurer.

In how many ways can these 3 positions be filled from the 8 farmers?

Apply the **product rule** step by step:

Position	Choices available
Chairperson	8 (any of the 8 farmers)
Secretary	7 (one farmer is now Chairperson)
Treasurer	6 (two farmers are now assigned)

$$\text{Total} = 8 \times 7 \times 6 = 336$$

Now generalise: selecting  $r$  positions from  $n$  people:

$$n \times (n - 1) \times (n - 2) \times \dots \times (n - r + 1)$$

## Writing It as a Factorial Ratio

$$\begin{aligned} & n \times (n - 1) \times \dots \times (n - r + 1) \\ & n \times (n - 1) \times \dots \times (n - r + 1) \times \underbrace{(n - r)!}_{\text{multiply and divide}} \\ = & \frac{\quad}{\underbrace{(n - r)!}_{\text{same thing}}} \end{aligned}$$

The numerator is just  $n!$  (the full factorial):

$$\boxed{{}^n P_r = \frac{n!}{(n - r)!}}$$

## Definition

$${}^n P_r = \frac{n!}{(n-r)!}$$

(read: “*n* permute *r*” or “*P* of *n*, *r*”) is the number of ways to **arrange** *r* objects chosen from *n* distinct objects, **where order matters**.

## Permutations — Order Matters!

### The Critical Insight

In the election example:

{**Amina = Chair, Baraka = Secretary**} is a **different** outcome from {**Baraka = Chair, Amina = Secretary**}.

The same two people — but different roles.

**Changing the order creates a different arrangement.**

This is the defining feature of a permutation: **order matters**.

## Special cases

Case	Formula	Meaning
All $n$ objects arranged	${}^n P_n = n!$	Every object used in every position
Choose 1 from $n$	${}^n P_1 = n$	$n$ ways to pick one
Choose 0 from $n$	${}^n P_0 = 1$	Only one way to arrange nothing

## Permutations — Step-by-Step Worked Examples

### Example 1: Farm Input Codes

A warehouse labels its storage rooms with a 3-letter code using letters from **{A, B, C, D, E}**, with no letter repeated.

How many distinct 3-letter codes are possible?

$n = 5$  letters,  $r = 3$  positions, order matters (ABC BAC)

$${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = \mathbf{60} \text{ codes}$$

### Example 2: Election of Officers

A farmers' cooperative of **12 members** must elect a **President, Vice-President, and Treasurer**.

$n = 12$ ,  $r = 3$ , order matters (different roles)

$${}^{12}P_3 = \frac{12!}{9!} = 12 \times 11 \times 10 = \mathbf{1,320} \text{ ways}$$

### Example 3: Farm Application — Scheduling

The officer must visit exactly **5 out of 9 farms** in a single day, in a specific route order.

$${}^9P_5 = \frac{9!}{4!} = 9 \times 8 \times 7 \times 6 \times 5 = \mathbf{15,120}$$
 possible routes

## Practice 5.3 — Permutations

### Your Turn

**(a)** A district agricultural show awards **1st, 2nd, and 3rd prizes** to farmers displaying livestock.

**20 farmers** have entered. In how many ways can the prizes be awarded?

**(b)** A farm manager wants to schedule visits to **4 out of 7 farms** in a single morning, visiting each farm in a specific order. How many different morning schedules are possible?

**(c)** How many different 4-digit PIN codes can be formed from the digits 1–9 if: (i) No digit may be repeated? (ii) Digits may be repeated? (*Note: repetition allowed means you cannot use the standard permutation formula — think about the product rule instead.*)

**(d)** Verify that  ${}^6P_6 = 6!$  using the formula  ${}^nP_r = n!/(n-r)!$ . Explain in words why this makes sense.

## 5.5 Combinations

## When Order Does NOT Matter

### A Different Problem

The cooperative needs to select **3 farmers** from 8 to form a **welfare committee**.

Unlike the election, the committee has **no ranks or roles**.

Being selected is what matters — not who is chosen first.

{Amina, Baraka, Chakula} is the **same committee** as {Baraka, Chakula, Amina}.

If we used the permutation formula, we would count each group of **3 multiple times** — once for every possible ordering of that same group.

How many times does each group of 3 get over-counted?

Exactly  $3! = 6$  times (the number of ways to arrange 3 people).

## Deriving the Combination Formula

**Start** from the permutation formula:  ${}^n P_r = \frac{n!}{(n-r)!}$  ( counts every ordered arrangement). **Each unordered group** of  $r$  items is counted  $r!$  times (once per arrangement of those  $r$  items).

**Fix the overcounting** — divide by  $r!$ :

$${}^n C_r = \frac{{}^n P_r}{r!} = \frac{n!}{(n-r)! \times r!}$$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

### Definition

${}^n C_r$  (read: “ $n$  choose  $r$ ”, also written  $\binom{n}{r}$ ) counts the number of ways to **select**  $r$  objects from  $n$  distinct objects **where order does not matter**.

# Visual: Why We Divide by $r!$

**.P. = 12 ordered pairs  $\rightarrow \div 2! = 2 \rightarrow$  .C. = 6 unordered groups**

Blue = one arrangement of the group. Red = the same group, reversed. Both count as ONE comb

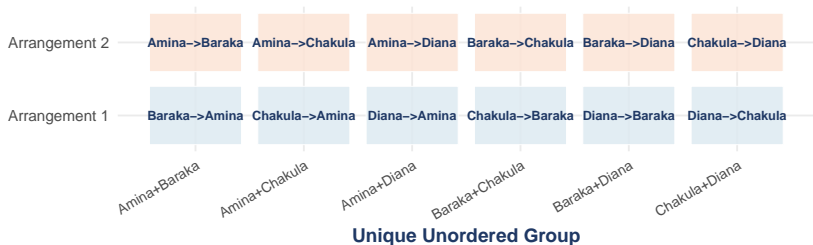


Figure 3

${}^4P_2 = 12 \text{ ordered pairs} \div 2! = 2 \text{ arrangements per pair} = {}^4C_2 = \mathbf{6}$   
combinations

## Verifying with the Formula

$${}^4C_2 = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{24}{2 \times 2} = \frac{24}{4} = \mathbf{6\checkmark}$$

**Back to the welfare committee problem ( $n = 8, r = 3$ ):**

$${}^8C_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = \frac{336}{6} = \mathbf{56} \text{ committees}$$

Compare with the permutation:  ${}^8P_3 = 336$ .

The combination is  $336 \div 6 = 56$  — exactly  $3! = 6$  times smaller, because each group of 3 was counted 6 times by the permutation formula.

## Properties of Combinations

These properties help you compute faster and check your answers:

### Property 1 — Symmetry:

$${}^n C_r = {}^n C_{n-r}$$

Choosing  $r$  to include is the same as choosing  $n - r$  to exclude.

$${}^{10} C_7 = {}^{10} C_3 = \frac{10!}{3!7!} = 120 \quad (\text{much easier to compute as } {}^{10} C_3)$$

### Property 2 — End cases:

$${}^n C_0 = 1 \quad {}^n C_n = 1$$

There is exactly one way to choose nothing, and one way to choose everything.

## Combinations — Worked Examples

### Example 1: Selecting Farms for a Pilot Programme

The district office wants to select **4 farms** from **10** to pilot a new irrigation scheme.

The farms are treated equally — no ranking.

$${}^{10}C_4 = \frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = \frac{5,040}{24} = \mathbf{210} \text{ possible selections}$$

### Example 2: Choosing a Sub-committee

From the welfare committee of 8, choose **2 members** to attend a national conference.

Use symmetry:  ${}^8C_2 = {}^8C_6$ .

$${}^8C_2 = \frac{8 \times 7}{2 \times 1} = \frac{56}{2} = \mathbf{28} \text{ ways}$$

## Practice 5.4 — Combinations

### Your Turn

**(a)** A research team selects **3 soil sample sites** from **12** possible locations across a farm.

The sites are treated equally.

How many different selections are possible?

**(b)** A cooperative has **15 members**. A committee of **5** is to be formed with no specific roles.

(i) How many committees are possible?

(ii) Use the symmetry property to verify:  ${}^{15}C_5 = {}^{15}C_{10}$ .

**(c)** Compute  ${}^7C_0$ ,  ${}^7C_1$ ,  ${}^7C_7$ . Interpret each result in plain language.

**(d)** A lottery draws **6 numbers** from **45**.

A farmer buys one ticket with 6 numbers.

How many possible winning combinations are there?

(This tells us how unlikely winning is!)

## 5.6 Permutations vs. Combinations — The Decision

# The Central Question

The One Question That Decides Everything

**“Does changing the order of the selected items give a NEW, DISTINCT outcome?”**

If **YES** → the arrangement matters → use **Permutation**

$$\binom{n!}{(n-r)!}$$

If **NO** → only the group matters → use **Combination**

$$\binom{n!}{r!(n-r)!}$$

## Practice 5.5 — Permutation or Combination?

### Your Turn

For each problem: (i) state whether to use P or C and why, (ii) write the correct expression, (iii) calculate the answer.

**(a)** 6 farmers line up for a group photograph. How many different orderings are possible?

**(b)** A quality inspector selects 4 grain bags from a batch of 15 for testing. How many different samples can be chosen?

**(c)** A district office must assign 3 agricultural extension officers to 3 different villages (Officer A to Village 1, Officer B to Village 2, etc.). How many assignments are possible?

**(d)** The same 3 officers are sent as a team to one village with no specific roles. How many teams are possible from the 8 available officers?

**(e)** Show numerically that your answer to (c) is exactly  $3!$  times your answer to (d).

Explain why this relationship holds.

# Keywords

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## Permutation words

arrange, order, rank  
schedule, route, sequence  
assign to *different* roles  
1st, 2nd, 3rd (prizes/positions)  
PIN, code, password

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## Combination words

select, choose, pick  
group, team, committee  
no specific roles  
sample, subset  
delegation, panel

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## 5.7 Applied Problems

## Counting Arrangements with Restrictions

Sometimes conditions restrict which arrangements are valid. We use the product rule together with permutations/combinations.

### Problem: A Committee Must Include a Specific Person

A cooperative of **10 members** must form a committee of **4**. The cooperative president **must be** on the committee.

**Step 1:** The president's place is fixed — 1 way to include them.

**Step 2:** Choose the remaining **3** members from the other **9**:

$${}^9C_3 = \frac{9!}{3!6!} = \frac{9 \times 8 \times 7}{6} = \frac{504}{6} = \mathbf{84} \text{ committees}$$

Compare: Without the restriction,  ${}^{10}C_4 = 210$  committees. The restriction reduces the count from 210 to **84**.

## Questions

### **Problem 1: Sports Selection (The Star Player)**

The JUCo football coach must choose a captaincy board of 3 players from a squad of 12 available players. The team's star striker must be on this board. How many different boards can be formed?

### **Problem 2: Lab Group Assignment (The Equipment Expert)**

A statistics lab session requires a group of 4 students to work on a data visualization project out of a class of 15 students. Because the project requires heavy programming, the student who knows Python must be included in this specific group. How many different group configurations are possible?

### **Problem 3: Employment Panel (The Department Head)**

An agricultural research institute in Morogoro is forming an interview panel of 5 experts from a pool of 9 senior scientists. The Head of the Agronomy Department must be on the panel. How many different panels can be created?

# Counting When a Specific Person Must NOT Be Included

## Problem: A Person Is Excluded

From the same 10 members, form a committee of 4 **excluding** one specific member.

**Approach:** Remove the excluded person — now choose 4 from **9**:

$${}^9C_4 = \frac{9!}{4!5!} = \frac{9 \times 8 \times 7 \times 6}{24} = \frac{3,024}{24} = \mathbf{126} \text{ committees}$$

## Questions

**Problem 1: Sports Selection (The Injured Player):** The JUCo football coach must choose a captaincy board of 3 players from a squad of 12 available players. The team's vice-captain is injured and cannot be included on this board. How many different boards can be formed?

**Problem 2: Lab Group Assignment (The Scheduling Conflict):** A statistics lab session requires a group of 4 students to work on a data visualization project out of a class of 15 students. One student has an exam conflict during that time and must be excluded from this group. How many different group configurations are possible?

**Problem 3: Employment Panel (The Conflict of Interest):** An agricultural research institute in Morogoro is forming an interview panel of 5 experts from a pool of 9 senior scientists. One scientist is related to a job applicant and cannot serve on the panel. How many different panels can be created?

## Arrangements of a Word

### Problem: Arrangements of a Word with Repeated Letters

How many distinct arrangements of the letters in **MAIZE** are there?

All 5 letters are distinct  $\rightarrow 5! = 120$

What about **GRASS**? Letters: G, R, A, S, S — the letter S repeats twice.

Dividing by  $2!$  removes the over-counting of identical S's:

$$\frac{5!}{2!} = \frac{120}{2} = \mathbf{60} \text{ arrangements}$$

## Questions

**Problem 1:** For an upcoming test review sheet, you want to scramble the key vocabulary term VARIANCE. How many unique letter arrangements can you generate?

**Problem 2:** A farmer is labeling storage bins for different crop varieties. How many distinct ways can the letters in the word CASSAVA be rearranged?

**Problem 3:** How many distinct arrangements of the letters in the word COMBINATIONS can be made?

## 5.8 Summary

# All Key Formulas

## Building blocks:

$$n! = n \times (n - 1) \times \dots \times 1$$

$$0! = 1$$

## Fundamental rules:

Sum Rule:  $m + n$

Product Rule:  $m \times n$

## Arrangements (order matters):

$${}^n P_r = \frac{n!}{(n - r)!}$$

## Selections (order does not matter):

$${}^n C_r = \frac{n!}{r!(n - r)!}$$

## Relationship:

$${}^n P_r = {}^n C_r \times r!$$



Tip

**The one question:** *Does swapping two selected items create a different outcome?*

YES  $\rightarrow$   ${}^n P_r$  | NO  $\rightarrow$   ${}^n C_r$

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6. **Combinations** — derived by dividing permutations by  $r!$ ; order does not matter;  ${}^n C_r = n! / r!(n - r)!$
7. **Relationship** —  ${}^n P_r = {}^n C_r \times r!$

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7. **Relationship** —  ${}^n P_r = {}^n C_r \times r!$
8. **Applications** — restrictions (must/must not include), repeated elements

## Comprehensive Practice Question

### Bringing It All Together

A cooperative of **15 farmers** must carry out the following tasks. For each, state the method, write the expression, and calculate.

**(a)** Elect a **President, Vice-President, and Secretary** (3 distinct roles from 15).

**(b)** Form a **welfare committee** of **4 members** (equal roles, from 15).

**(c)** The welfare committee must include the **oldest farmer** and the **youngest farmer**.

How many such committees are possible?

**(d)** Form a **project team of 3** from the remaining 12 farmers (after the officers are elected).

**(e)** The project team visits **5 farms** in a chosen order. The team selects 5 farms from **12** available farms and decides on a visit sequence.

How many complete schedules (farm + sequence) are possible?

**(f)** Calculate  ${}^{15}C_4 \div {}^{15}C_{11}$  and explain the result using the

## Next Topic

### Topic VI: Probability Theory

*Classical Probability · Addition Rules*

*Multiplication Rules · Conditional Probability*

*Bayes' Theorem · Expected Value*



#### The direct connection:

$$P(A) = \frac{\text{number of ways A can occur}}{\text{total possible outcomes}}$$

Both numbers in this fraction will be calculated using  ${}^n P_r$  and  ${}^n C_r$ .

Everything in Topic V was preparation for Topic VI.

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# Thank You

## Questions?

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*“Before you can ask ‘how likely?’  
you must first answer ‘how many?’ ”*