QMS 101 Introductory Statistics

Topic VII: Probability Distributions (Simplified)

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Learning Outcomes

By the end of this topic, you will be able to:

- Explain what a **probability distribution** is
- Define and use the **Binomial distribution**
- Define and use the **Poisson distribution**
- Define and use the **Normal distribution**
- Read the **Z-table** correctly, step by step

7.1 What Is a Probability Distribution?

Key Definitions

Probability Distribution

A list or model showing **every possible value** a variable can take, together with the **probability of each value** occurring. All the probabilities must add up to **1**.

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Random Variable (X)

A quantity whose value is determined by chance — e.g. the number of damp bags in a delivery, or the weight of a sack of grain.

Discrete vs. Continuous

Discrete Random Variable

Can only take **countable, separate** values (0, 1, 2, 3, ...).
Example: number of defective seeds.

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Example: weight of a grain sack (e.g. 49.7 kg).

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Example: weight of a grain sack (e.g. 49.7 kg).

Feature	Discrete	Continuous
Values	Countable	Any value in a range
Distributions	Binomial, Poisson	Normal
Farm example	No. of damp bags	Weight of a harvest bag

7.2 The Binomial Distribution

Definition

Binomial Distribution

A probability model used when an experiment is repeated a **fixed number of times**, each repeat has only **two possible outcomes** (success or failure), and the chance of success stays the **same** every time. **Examples**

- Flip a coin 10 times to count how many times it lands on heads.
- Quality control: A factory tests (50) lightbulbs from a batch to see how many are defective (success) versus functional (failure).
- Medical trials: A doctor treats (20) patients with a new drug to count how many are cured versus not cured.
- E-commerce sales: An online store tracks (100) website visitors to see how many buy an item versus leave without purchasing.

Key Terms

Terms You Must Know

- **Trial** — one repetition of the experiment (e.g. inspecting one bag)
- **Success** — the outcome we are counting (e.g. "bag is damp")
- n — the fixed number of trials
- p — the probability of success on a single trial
- q — the probability of failure, $q = 1 - p$
- X — the number of successes out of n trials

When to Use It — the BINS Conditions

Check All Four Before You Apply the Formula

B — Binary outcomes (success/failure)
I — Independent trials
N — Number of trials fixed in advance
S — Same probability p every trial
If any one condition fails, do **not** use the Binomial distribution.

The Formula (Use It — No Need to Derive It)

$$\boxed{P(X = x) = \binom{n}{x} p^x \, q^{n-x}}$$

$$\mu = np \qquad \sigma^2 = npq \qquad \sigma = \sqrt{npq}$$

What Each Symbol Does

$\binom{n}{x}$ counts **how many ways** x successes can occur (from Topic V).
p^x is the chance all x successes happen.
q^{n-x} is the chance all remaining trials are failures.

Example 1 — Grain Inspection

What the Question Requires

A batch of bags has a **20% chance** each bag is damp ($p = 0.20$). An inspector checks $n = 10$ bags.
Find: $P(X = 0)$ — the probability that **none** of the 10 bags are damp.
What we need: identify n, p, q, and x, then substitute directly into the formula.

Solution

Step 1 — Identify the values: $n = 10$, $p = 0.20$, $q = 0.80$, $x = 0$

Step 2 — Substitute into the formula:

$$P(X = 0) = \binom{10}{0}(0.20)^0(0.80)^{10} = 1 \times 1 \times 0.1074 = \mathbf{0.1074}$$

Step 3 — Find the mean and SD:

$$\mu = np = 10 \times 0.20 = 2 \text{ damp bags expected}$$

$$\sigma = \sqrt{npq} = \sqrt{10 \times 0.20 \times 0.80} = \mathbf{1.265}$$

Example 2 — Loan Approval

What the Question Requires

A microfinance officer knows that **65% of loan applications** are approved ($p = 0.65$). She reviews $n = 6$ applications in a day.
Find: $P(X = 4)$ — the probability that **exactly 4** of the 6 applications are approved.
What we need: the same BINS check, then substitute $x = 4$ into the formula.

Solution

Step 1 — Identify the values: $n = 6$, $p = 0.65$, $q = 0.35$, $x = 4$

Step 2 — Substitute into the formula:

$$P(X = 4) = \binom{6}{4}(0.65)^4(0.35)^2 = \mathbf{0.328}$$

Step 3 — Find the mean and SD:

$$\mu = np = 6 \times 0.65 = 3.9 \text{ approvals expected}$$

$$\sigma = \sqrt{npq} = \sqrt{6 \times 0.65 \times 0.35} = \mathbf{1.168}$$

Practice 7.1 — Binomial Distribution

Your Turn

A farmer plants **8 seeds** of a new maize variety. Each seed has a **70% germination rate** ($p = 0.70$). Let X = number of seeds that germinate.

(a) Confirm the BINS conditions hold for this problem.
(b) Find $P(X = 8)$ — all seeds germinate.
(c) Find $P(X = 5)$.
(d) Calculate μ, σ^2, and σ, and interpret μ in one sentence.

7.3 The Poisson Distribution

Definition

Poisson Distribution

A tool for predicting the **exact number of random arrivals** (like customers, phone calls, or website clicks) that will happen in a **set period of time**, based solely on your **historical average**.

Core features

- **Predicts event frequency**: Estimates how many times an event will happen within a fixed timeframe or location.
- **Driven by averages**: Relies on knowing only the **historical average rate** of occurrence.
- **Emphasizes randomness**: Applied when individual events occur completely at random.

Essential rules

- **Independent events**: One occurrence never influences the likelihood of the next.
- **No simultaneous events**: Occurrences must happen one at a time, never perfectly together.
- **Constant rate**: The baseline average frequency stays steady throughout the observed period.

Examples

- **Website traffic**: Counting the number of visitors hitting a homepage every minute.
- **Hospital ERs**: Tracking patient arrivals between midnight and 2:00 AM.
- **Retail management**: Predicting how many shoppers enter a grocery lane each hour.

Key Terms

Terms You Must Know

- λ **(lambda)** — the average (mean) number of occurrences in the given interval
- **Interval** — the fixed unit being studied (e.g. per week, per hectare, per hour)
- X — the actual number of occurrences observed in that interval

The Formula (Use It — No Need to Derive It)

$$\boxed{P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}}$$

$$\mu = \lambda \qquad \sigma^2 = \lambda \qquad \sigma = \sqrt{\lambda}$$

Special Property

The **mean always equals the variance** ($\mu = \sigma^2 = \lambda$). This is unique to the Poisson distribution — no other common distribution shares it.

Example 1 — Equipment Breakdowns

What the Question Requires

A farm records an average of $\lambda = 3$ equipment breakdowns per month.
Find: $P(X = 0)$ — the probability of **no breakdowns** in a given month.
What we need: identify λ and x, then substitute into the formula.

Solution

Step 1 — Identify the values: $\lambda = 3$, $x = 0$

Step 2 — Substitute into the formula:

$$P(X = 0) = \frac{e^{-3} \times 3^0}{0!} = \frac{0.0498 \times 1}{1} = \mathbf{0.0498}$$

Step 3 — Mean and SD:

$$\mu = \lambda = 3 \qquad \sigma = \sqrt{3} = \mathbf{1.73}$$

Example 2 — Customer Complaints

What the Question Requires

A cooperative store receives an average of $\lambda = 4$ complaints per week.
Find: $P(X = 2)$ — the probability of **exactly 2 complaints** in a given week.
What we need: identify λ and x, then substitute directly.

Solution

Step 1 — Identify the values: $\lambda = 4$, $x = 2$

Step 2 — Substitute into the formula:

$$P(X = 2) = \frac{e^{-4} \times 4^2}{2!} = \frac{0.0183 \times 16}{2} = \mathbf{0.1465}$$

Step 3 — Mean and SD:

$$\mu = \lambda = 4 \qquad \sigma = \sqrt{4} = \mathbf{2.0}$$

Practice 7.2 — Poisson Distribution

Your Turn

A veterinary clinic treats an average of $\lambda = 5$ emergency cases per day.

(a) State the conditions that must hold for the Poisson distribution to apply here.

(b) Find $P(X = 0)$ — no emergencies in a day.

(c) Find $P(X = 3)$.

(d) Calculate μ, σ^2, and σ. Explain in one sentence why $\mu = \sigma^2$ here.

Binomial vs. Poisson — Quick Comparison

Feature	Binomial	Poisson
Number of trials	Fixed (n)	No fixed limit
What you need to know	n and p	Average rate λ
Mean	np	λ
Variance	npq	λ
Typical use	Fixed sample checked for defects	Rare events over time/space

7.4 The Normal Distribution

Definition

Normal Distribution

A continuous, symmetric, **bell-shaped** probability distribution. Fully described by just two numbers: its mean (μ) and standard deviation (σ). Written as $X \sim N(\mu, \sigma^2)$.

Key Terms

Terms You Must Know

- **Symmetric** — the left half of the curve is a mirror image of the right half
- **Mean = Median = Mode** — all three sit exactly at the centre of the curve
- **Bell-shaped** — most values cluster near the mean; fewer occur further away
- **Standard Normal Distribution** — a special Normal distribution with $\mu = 0$ and $\sigma = 1$; all Normal problems are converted to this form before using the Z-table
- **Z-score (standard score)** — the number of standard deviations a value X lies above or below the mean

The Z-score Formula

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$

- Z: Number of standard deviations away from the mean
- X: Your specific raw data point value
- μ: The overall historical average (mean)
- σ: The spread or volatility of the data (standard deviation)

Z-score	Meaning
$Z = 0$	Value is exactly at the mean
$Z = +1$	Value is 1 SD **above** the mean
$Z = -1.5$	Value is 1.5 SDs **below** the mean

The Empirical Rule

Empirical Rule (68–95–99.7 Rule)

For any Normal distribution:

- About **68%** of values lie within $\mu \pm \sigma$
- About **95%** of values lie within $\mu \pm 2\sigma$
- About **99.7%** of values lie within $\mu \pm 3\sigma$

7.5 How to Read the Z-Table

What the Z-Table Gives You

Z-table (Standard Normal Table)

A table giving the probability (area under the curve) to the **left** of a given Z-score, i.e. $P(Z \leq z)$.

Extract of a Z-table:

z	.00	.01	.02	.03	.04	.05
0.0	.5000	.5040	.5080	.5120	.5160	.5199
0.5	.6915	.6950	.6985	.7019	.7054	.7088
1.0	.8413	.8438	.8461	.8485	.8508	.8531
1.5	.9332	.9345	.9357	.9370	.9382	.9394
2.0	.9772	.9778	.9783	.9788	.9793	.9798

The Six Steps

Follow These Every Time

Step 1 — Convert X to a Z-score: $Z = \dfrac{X - \mu}{\sigma}$, rounded to 2 decimal places.
Step 2 — Split the Z-score: first digit + 1 decimal $\rightarrow$ **row**; 2nd decimal $\rightarrow$ **column**.
Step 3 — Find the **row** in the left-hand column (e.g. "1.2").
Step 4 — Find the **column** along the top row (e.g. ".05").
Step 5 — Read off the value where row and column meet: this is $P(Z \leq z)$.
Step 6 — Adjust for what the question actually asks (see table below).

Adjusting the Table Value

You want	Formula
$P(Z \leq z)$	Read directly from the table
$P(Z > z)$	$1 - P(Z \leq z)$
$P(Z \leq -z)$ (negative Z)	$1 - P(Z \leq z)$ (by symmetry)
$P(z_1 < Z < z_2)$	$P(Z \leq z_2) - P(Z \leq z_1)$

Example 1 — Direct Lookup

What the Question Requires

Find $P(Z \leq 1.25)$.
What we need: locate row 1.2 and column .05 in the Z-table — no further adjustment needed since the question already asks for $P(Z \leq z)$.

Solution

Step 1: $z = 1.25$ (already a Z-score)
Step 2: Row = 1.2, Column = .05
Step 3–5: Row "1.2" meets column ".05" at **0.8944**
Step 6: No adjustment needed (question asks for $P(Z \leq z)$ directly)

$$P(Z \leq 1.25) = \mathbf{0.8944}$$

Example 2 — Real Data, Upper Tail

What the Question Requires

Crop yields are Normal with $\mu = 50$ bags/acre, $\sigma = 8$ bags/acre.
Find: $P(X > 62)$ — the probability a farm's yield **exceeds 62** bags/acre.
What we need: convert X to a Z-score first (Step 1), then look up the table value, then adjust because the question asks for "greater than" (Step 6).

Solution

Step 1: $Z = \dfrac{62 - 50}{8} = 1.50$

Step 2–5: Row "1.5", column ".00" $\rightarrow P(Z \leq 1.50) = 0.9332$

Step 6: Question asks for $P(Z > z)$, so subtract from 1:

$$P(X > 62) = 1 - 0.9332 = \mathbf{0.0668}$$

Interpretation

Only about **6.7%** of farms are expected to yield more than 62 bags/acre.

The Greater-Than Case

Core Concept

When finding the probability of a value being "greater than" ($P(X > x)$), the table gives the area to the left. You must subtract the table value from 1 to find the area to the right.

Problem: A machine packs sugar into bags with $\mu = 10$ kg and $\sigma = 0.5$ kg. Find $P(X > 10.75)$.

Solution

- **Step 1**: $Z = \dfrac{10.75 - 10}{0.5} = \dfrac{0.75}{0.5} = 1.50$
- **Step 2–5**: Row **1.5**, column **.00** $\rightarrow P(Z \leq 1.50) = 0.9332$
- **Step 6**: Apply the upper-tail rule:

$$P(X > 10.75) = 1 - 0.9332 = \mathbf{0.0668}$$

Interpretation

Calculations indicate a 6.68% probability of selecting a bag over 10.75 kg.

The Negative Z-Score Case

Core Concept

Standard Z-tables only list positive Z-scores. By symmetry, the area to the left of a negative Z-score ($P(Z \leq -z)$) is exactly equal to the area to the right of its positive counterpart ($1 - P(Z \leq z)$).

Problem: Using the same sugar packing machine ($\mu = 10$ kg, $\sigma = 0.5$ kg), find $P(X < 9.00)$.

Solution

- **Step 1**: $Z = \dfrac{9.00 - 10}{0.5} = \dfrac{-1.00}{0.5} = -2.00$
- **Step 2–5**: Look up positive value $Z = 2.00 \rightarrow$ Row **2.0**, column **.00** $\rightarrow 0.9772$
- **Step 6**: Apply the symmetry rule for a negative Z-score:

$$P(X < 9.00) = 1 - 0.9772 = \mathbf{0.0228}$$

Interpretation

Calculations indicate a 2.28% probability of selecting one under 9.00 kg.

The Between Case

Core Concept

To find the probability between two values ($P(x_1 < X < x_2)$), calculate the Z-scores for both boundaries, look up their probabilities, and subtract the smaller probability from the larger probability.

Problem: Using the same sugar packing machine ($\mu = 10$ kg, $\sigma = 0.5$ kg), find $P(10.00 < X < 10.26)$.

Solution

- **Step 1**: Convert both boundary values.
  - $Z_1 = \dfrac{10.00 - 10}{0.5} = 0.00$
  - $Z_2 = \dfrac{10.26 - 10}{0.5} = 0.52$
- **Step 2–5**: Lookup $Z_1 = 0.00 \to \mathbf{0.5000}$ and $Z_2 = 0.52 \to$ Row **0.5**, col **.02** $\to \mathbf{0.6985}$

Solution ...

- **Step 6**:
  $P(10.00 < X < 10.26) = 0.6985 - 0.5000 = \mathbf{0.1985}$

Interpretation

Calculations indicate a 19.85% probability for weights between 10.00 kg and 10.26 kg.

Practice 7.3 — Normal Distribution & Z-Table

Your Turn

The weight of bags of grain filled at a cooperative store is Normal with $\mu = 50$ kg and $\sigma = 2$ kg.

(a) Find $P(X > 53)$ — a bag is overfilled beyond 53 kg.

(b) Find $P(X < 47)$ — a bag is underfilled below 47 kg.

(c) Find $P(48 < X < 54)$ — a bag falls within this acceptable range.

(d) For each part, show your Z-score calculation and state clearly which of the six steps you used to adjust the table value (if any).

7.6 Summary

All Three Distributions — At a Glance

Distribution	Type	What it needs	Mean	Variance
Binomial	Discrete	Fixed trials n, probability p	np	npq
Poisson	Discrete	Average rate λ	λ	λ
Normal	Continuous	Mean μ, SD σ	μ	σ^2

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4. **Normal distribution** — continuous, symmetric, bell-shaped, defined by μ and σ

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1. **Probability distributions** — a full model of outcomes and their probabilities
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3. **Poisson distribution** — rare events over a fixed interval, known average rate
4. **Normal distribution** — continuous, symmetric, bell-shaped, defined by μ and σ
5. **Z-scores** — convert any Normal value into standard deviations from the mean and **Reading the Z-table** to find any probability

Comprehensive Practice — Bringing It All Together

Final Practice

(a) A batch of 10 solar lamps has a 15% chance each lamp is faulty ($p = 0.15$). Find $P(X = 2)$ using the Binomial distribution, and state μ and σ.

(b) A clinic records an average of $\lambda = 6$ walk-in patients per hour. Find $P(X = 4)$ using the Poisson distribution.

(c) Delivery times for an online store are Normal with $\mu = 4$ days and $\sigma = 0.8$ days. Find $P(X > 5.5)$, showing your Z-score and all six Z-table steps.

(d) For part (c), find the delivery time below which only **10%** of orders arrive (i.e. find X such that $P(X < x_0) = 0.10$).

Thank You

Questions?

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*"The average tells you where the centre is.
The standard deviation tells you how much to trust it.
The Z-table tells you exactly how rare or common any value really is."*