QMS 101 Introductory Statistics

Tutorial Sheet — Topic VII: Probability Distributions

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Instructions

- Attempt every question on your own first, showing full working.
- State clearly which distribution (Binomial, Poisson, or Normal) applies to each question, and why.
- For Normal distribution questions, always show your Z-score calculation and state which of the six Z-table steps you are applying.
- A Z-table extract is provided at the end of this sheet for reference.

Section A: Understanding Probability Distributions

Question 1

For each scenario below, state whether the random variable X is **discrete** or **continuous**, and briefly justify your answer.

(a) The number of customers who visit a shop in an hour.
(b) The height of maize plants in a field (cm).
(c) The number of defective light bulbs in a box of 20.
(d) The time taken (in minutes) for a bus to arrive.
(e) The number of goals scored in a football match.
(f) The weight of a sack of rice (kg).

Question 2

A quality officer records the number of scratches found on 50 inspected phone screens:

Scratches (x)	0	1	2	3	4
Frequency	18	16	10	4	2

(a) Convert this into a probability distribution table. Verify that $\sum P(x) = 1$.

(b) Calculate $E(X)$, $\text{Var}(X)$, and $\text{SD}(X)$.

(c) Interpret $E(X)$ in the context of phone screen inspections.

Section B: The Binomial Distribution

Question 3

A multiple-choice quiz has 10 questions, each with 4 possible answers (only one correct). A student guesses randomly on every question.

(a) State the values of n and p for this situation.

(b) Confirm that the BINS conditions are satisfied.

(c) Find the probability the student gets **exactly 3** questions correct.

(d) Find the probability the student gets **none** correct.

(e) Calculate the expected number of correct answers, μ, and the standard deviation σ.

Question 4

A seed company claims that **85%** of its bean seeds germinate. A farmer plants **12** seeds.

(a) Let X = number of seeds that germinate. State the distribution of X and its parameters.

(b) Find $P(X = 12)$ — all seeds germinate.

(c) Find $P(X = 10)$.

(d) Find $P(X \leq 1)$ — at most 1 seed germinates. *(Hint: this should be a very small probability — if your answer is large, check your working.)*

(e) Calculate μ, σ^2, and σ, and interpret μ for the farmer.

Question 5

A call centre finds that **40%** of incoming calls result in a completed sale. In a sample of **7** calls:

(a) Find the probability that **exactly 4** calls result in a sale.

(b) Find the probability that **at least 5** calls result in a sale. *(Hint: calculate $P(5)+P(6)+P(7)$.)*

(c) Find the probability that **at most 2** calls result in a sale.

(d) Calculate the mean and standard deviation of the number of sales per 7 calls.

Section C: The Poisson Distribution

Question 6

A rural health clinic receives an average of **6 patients per hour** during flu season.

(a) State the value of λ and explain what it represents.

(b) Find $P(X = 0)$ — no patients arrive in a given hour.

(c) Find $P(X = 6)$ — exactly the average number arrive.

(d) Find $P(X \leq 3)$.

(e) Find $P(X \geq 8)$. *(Hint: use the complement rule with $P(X \leq 7)$.)*

(f) Calculate μ, σ^2, and σ. What do you notice about μ and σ^2?

Question 7

A textile factory finds an average of **2 flaws per 100 metres** of fabric produced.

(a) Explain why the Poisson distribution is appropriate here rather than the Binomial.

(b) Find the probability that a randomly selected 100-metre roll has **no flaws**.

(c) Find the probability that a roll has **exactly 2 flaws**.

(d) Find the probability that a roll has **more than 3 flaws**.

(e) A quality manager wants to know: if 500 metres of fabric are produced, what is the *new* value of λ for that length? Recalculate $P(X = 0)$ for the 500-metre length.

Section D: The Normal Distribution and the Z-Table

Question 8 — Z-Table Practice (Direct Lookup)

Using the Z-table provided, find the following probabilities. For each, state whether you read the value **directly** or needed to **adjust** it (and how).

(a) $P(Z \leq 0.84)$

(b) $P(Z \leq 2.03)$

(c) $P(Z > 1.15)$

(d) $P(Z \leq -0.67)$

(e) $P(Z > -1.40)$

(f) $P(-1.00 < Z < 1.50)$

(g) $P(0.25 < Z < 1.75)$

Question 9

The heights of maize plants in a large field are Normally distributed with $\mu = 165$ cm and $\sigma = 12$ cm.

(a) Find the Z-score for a plant that is **189 cm** tall.

(b) Find $P(X > 189)$.

(c) Find $P(X < 150)$.

(d) Find $P(150 < X < 180)$.

(e) A plant is considered "stunted" if its height falls in the bottom 5% of the distribution. Find the height cutoff for a stunted plant. *(Hint: find z_0 such that $P(Z < z_0) = 0.05$, then use $X = \mu + z_0\sigma$.)*

Question 10

Weekly wages at a small factory are Normally distributed with a mean of $\mu = TZS$ 85,000 and a standard deviation of $\sigma = TZS$ 9,000.

(a) Find the probability that a randomly selected worker earns **more than TZS 100,000** per week.

(b) Find the probability that a worker earns **less than TZS 70,000** per week.

(c) Find the probability that a worker earns **between TZS 75,000 and TZS 95,000** per week.

(d) Management wants to give a bonus to the top **15%** of earners. Find the minimum weekly wage needed to qualify for the bonus.

(e) Using the Empirical Rule (not the Z-table), state the range of wages within which approximately **95%** of workers fall.

Appendix: Z-Table Extract

Use this table for all Normal distribution questions. Values give $P(Z \leq z)$.

z	.00	.01	.02	.03	.04	.05	.06	.07
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808

Reminder: For negative Z-values, use symmetry: $P(Z \leq -z) = 1 - P(Z \leq z)$.

Common $e^{-\lambda}$ values for Poisson calculations:

λ	$e^{-\lambda}$	λ	$e^{-\lambda}$
1	0.3679	5	0.0067
2	0.1353	6	0.0025
3	0.0498	7	0.0009

λ	$e^{-\lambda}$	λ	$e^{-\lambda}$
4	0.0183	8	0.0003

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