QMS 101 Introductory Statistics

Tutorial Sheet — Topic VI: Probability Theory

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Section A: Basic Probability

Question 1

A bag contains **20 fruits**: 8 mangoes, 7 oranges, and 5 bananas. One fruit is picked at random.

(a) Write down the sample space.

(b) Find:

- (i) $P(\text{mango})$
- (ii) $P(\text{not a mango})$
- (iii) $P(\text{orange or banana})$

(c) Verify that your answers to (b)(i) and (b)(ii) sum to 1.

(d) A student says: *"There are 3 fruit types, so $P(\text{mango}) = \frac{1}{3}$."* Explain why this is incorrect.

Question 2

Out of **250 past growing seasons** in a district, **175 seasons** had adequate rainfall.

(a) Using the empirical (relative frequency) approach, find the probability that the next season has adequate rainfall.

(b) State **one** assumption this approach relies on.

(c) An agricultural officer with 30 years of experience says: *"I believe there is roughly an 80% chance of adequate rainfall this year, based on the early signs I am seeing."* Which of the three approaches to probability is this — classical, empirical, or subjective? Explain.

Section B: Types of Events

Question 3

A poultry farmer classifies each chicken in a flock as either **healthy** or **sick**. Separately, each chicken is also classified by **age**: chick, grower, or layer.

(a) Are "healthy" and "sick" mutually exclusive? Explain.

(b) Are "chick" and "grower" mutually exclusive? Explain.

(c) Suppose 200 chickens are surveyed: 40 are chicks, 90 are growers, and 70 are layers. Are these three age categories collectively exhaustive? Justify your answer using the numbers given.

(d) A farmer randomly inspects one chicken from the Northern pen and, separately, one chicken from a Southern pen 50 km away. Are the health statuses of these two chickens likely to be independent or dependent? Explain your reasoning.

Question 4

A box contains **6 light bulbs**: 2 are faulty and 4 are working. Two bulbs are selected one after another **without replacement**.

(a) Find $P(\text{first bulb is faulty})$.

(b) Find $P(\text{second bulb is faulty} \mid \text{first bulb was faulty})$.

(c) Are the two draws independent or dependent? Justify your answer using your results from (a) and (b).

Section C: Addition Rule and Multiplication Rule

Question 5

A district survey of **80 farms** found: 35 use **certified seeds (C)**, 28 use **organic manure (O)**, and 12 use **both**.

(a) Are C and O mutually exclusive? Justify with numbers.

(b) Find $P(C \cup O)$ — the probability a farm uses certified seeds or organic manure (or both). Show the addition rule clearly.

(c) Find the probability that a farm uses **neither** certified seeds nor organic manure.

(d) Find the probability that a farm uses certified seeds **only** (not organic manure).

Question 6

Two independent pest-control treatments are applied to two different farms.

- Farm 1 has a 90% chance the treatment succeeds: $P(\text{success}_1) = 0.90$
- Farm 2 has an 85% chance the treatment succeeds: $P(\text{success}_2) = 0.85$

(a) Find the probability that **both** treatments succeed.

(b) Find the probability that **both** treatments fail.

(c) Find the probability that **at least one** treatment succeeds.

(d) Find the probability that **exactly one** treatment succeeds.

Question 7

A sack contains **12 bags of rice**: 4 are understocked (underweight) and 8 are correctly filled. Two bags are drawn **without replacement**.

(a) Find $P(\text{both bags are understocked})$.

(b) Find P(both bags are correctly filled).

(c) Find P(at least one bag is understocked).

(d) Compare your answer in (a) to what the answer would be **with replacement**. Which is larger, and why?

Section D: Mixed Application Problems

Question 8

A cooperative has **100 members**. Of these, 55 attended a recent training workshop (T), and 40 members reported using improved storage techniques (S). 25 members did **both** — attended training and use improved storage.

(a) Construct a two-way (contingency) table summarising this information, including row and column totals.

(b) Find $P(T)$, $P(S)$, and $P(T \cap S)$.

(c) Find $P(T \cup S)$.

(d) Find the probability that a randomly selected member did **neither** attend training **nor** use improved storage.

(e) Two members are selected at random, **with replacement**. Find the probability that both attended training.

Question 9

A district agricultural office must:

1. Elect a **Chairperson, Secretary, and Treasurer** from **10 members** (three distinct roles).
2. Separately form an unranked **task force of 4** from the same 10 members.

(a) Calculate the number of ways to fill the three officer positions.

(b) Calculate the number of possible task forces of 4.

(c) A box contains 10 tickets, one for each of the 10 members. Two tickets are drawn at random without replacement to award a "Most Improved Farmer" prize and a "Community Service" prize (two distinct prizes — the same person cannot win both). How many different outcomes are possible?

(d) From the same 10 members, one farm is randomly chosen to host a field visit. There is a 30% chance the visit happens on a rainy day, and a 70% chance it happens on a dry day. If it is rainy, there is a 0.80 probability the visit is postponed. If it is dry, there is only a 0.05 probability the visit is postponed. Find the overall probability that the visit is postponed. *(Hint: this combines ideas from Topic VI's multiplication and addition rules — break into rainy and dry scenarios.)*

Section E: Expected Value

Question 10

A trader buys a perishable batch of tomatoes for resale. Based on past experience, the trader's profit depends on demand:

Demand Level	Probability	Profit (TZS '000)
High	0.30	60
Medium	0.45	35
Low	0.25	−10

(a) Verify that the probabilities sum to 1.

(b) Calculate the expected profit $E(X)$. Show a full working table.

(c) Calculate $E(X^2)$.

(d) Calculate $\text{Var}(X)$ and $\text{SD}(X)$.

(e) A second trader has a guaranteed profit of TZS 30,000 with certainty (no risk at all). Compare the two traders using expected value and standard deviation, and advise a risk-averse investor which option to choose.

Question 11

A motorbike-taxi (bajaji) operator is deciding whether to purchase comprehensive insurance for his vehicle for the year.

Without insurance, the possible outcomes over the year are:

Event	Probability	Cost to Operator (TZS '000)
No accident	0.75	0
Minor accident	0.20	−150
Major accident	0.05	−900

Insurance costs a flat **TZS 60,000** for the year and covers all accident costs completely (the operator pays nothing extra if an accident occurs).

(a) Calculate the expected cost **without** insurance.

(b) Calculate the expected cost **with** insurance. *(Hint: with insurance, the cost is always exactly TZS −60,000, regardless of what happens.)*

(c) Based on expected value alone, which option has the lower expected cost?

(d) Calculate $\text{SD}(X)$ for the uninsured case. What does this number tell the operator that the expected value in (a) does not?

(e) Give one practical reason the operator might still choose to buy insurance, even if the expected cost is higher without it.